Solutions to Probe 2012:

Part I:

1. 8:15 pm
2. 120
3. $\frac{12}{21}$
4. 2.22 miles
5. $-\frac{b}{c}$
6. 8
7. 9

Part II

1. $\frac{300}{7}$ miles/hr $\approx 42.8 \frac{m}{hr}$
2. 8 diamonds
3. 34 triangles

2012 PROBE Exam, Part I (no partial credit)

NAME: Key

1. Two cuckoo clocks were brought to the Tick Tock Clock Shop for repairs. Both clocks always told the correct time. However, one cuckoo came out every 45 minutes and the other came out every 55 minutes. One day, both cuckoos came out at 12.00 noon. When was the next time both cuckoos came out together?

   Answer: 8:15 pm

2. There are just enough microscopes, test tubes and calculators so that every 3 students have to share a microscope, every 4 students have to share a test tube and every 5 students had to share a calculator. The sum of microscopes, test tubes and calculators is 94. How many students are there?

   Answer: 120

3. A 6 sided die having its side numbered from 1 till 6 is tricked such that the probability of appearence of each side is proportional to its number. What is the probability of the event "Even"?

   Answer: $\frac{12}{21}$

4. In a ten-mile race, First beats Second by 1 mile and First beats Third by 3 miles. If the runners maintain constant speed throughout the race, by how many miles does Second beat Third?

   Answer: 2.22 miles

5. The sum of the reciprocals of the roots of the equation $ax^2 + bx + c = 0$ is

   Answer: $-\frac{b}{c}$

6. In the base ten number system the number 526 means $5 \cdot 10^2 + 2 \cdot 10 + 6$. In Fantasialand, however, numbers are written in the base r. Jones purchases an automobile for 440 monetary units (abbreviated m.u.). He gives the salesman a 1000 m.u. bill, and receives, in change, 340 m.u. The base r is

   Answer: 8

7. One side of a given triangle is 18 inches. Inside the triangle a line segment is drawn parallel to this side forming a trapezoid whose area is three-fourth of that of the triangle. The length of this segment, in inches, is:

   Answer: 9

1. 1st every 45 min
2nd every 55 min, need lcm = least common multiple

$\text{lcm}(45,55) = 5 \cdot \text{lcm}(9,11) = 5 \cdot 99 = 495$

$\Rightarrow$ after 495 min, both will come out again

$495 = 480 + 15 = 60 \cdot 8 + 15$, so 8:15 pm

2. $m + t + c = 94$

$s = 3m = 4t = 5c \Rightarrow m = \frac{s}{3}, t = \frac{s}{4}, c = \frac{s}{5}$

$\Rightarrow \frac{s}{3} + \frac{s}{4} + \frac{s}{5} = 94 \quad | \cdot 60$

$20s + 15s + 12s = 60 \cdot 94 = 5640$

$47s = 5640$

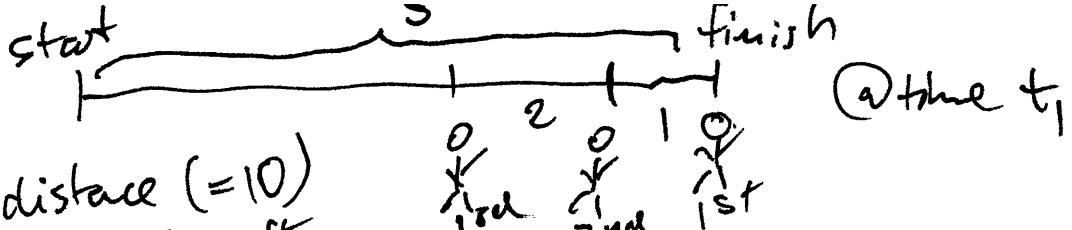
$s = \frac{5640}{47} = 120$

3. total # of dots $= 1+2+3 \ldots +6 = \frac{7 \cdot 6}{2} = 21$

$P(\text{"1"}) = \frac{1}{21}$, $P(\text{"2"}) = \frac{2}{21}$ etc

$P(\text{"2" or "4" or "6"}) = \frac{2}{21} + \frac{4}{21} + \frac{6}{21} = \frac{12}{21}$

4:



s = distance (=10)
a = speed of 1st runner
b = speed of 2nd runner
c = speed of 3rd runner

$10 = a \cdot t_1$ $\quad s = a \cdot t_1$ $\quad \frac{s}{t_1} - \frac{1}{t_1} = b$

$9 = b \cdot t_1$ $\quad s-1 = b \cdot t_1$

$7 = c \cdot t_1$ $\quad s-3 = c \cdot t_1$ $\quad -) \ \frac{s}{t_1} - \frac{3}{t_1} = c$

$\frac{-1}{t_1} + \frac{3}{t_1} = b - c$

2nd runner will finish 1 mile in $\frac{1}{b}$ time

That time, 2nd will advance his leadover 3rd by

$$\frac{1}{b} \cdot (b-c) = \frac{1}{b}\left(-\frac{1}{t_1} + \frac{3}{t_1}\right) \text{ miles}$$

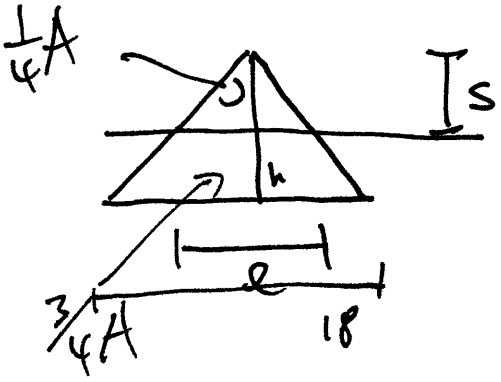
but $t_1 = \frac{s-1}{b}$

So, the overall lead of 2nd over 3rd at the time 2nd finishes is

$$2 + \frac{1}{b}(b-c) = 2 + \frac{1}{b}\left(-\frac{b}{s-1} + \frac{3b}{s-1}\right)$$

$$= 2 + \frac{1}{b}\,\frac{2b}{s-1} = 2 + \frac{2}{s-1} = 2 + \frac{2}{9} = \frac{20}{9} \approx \underline{2.22}$$

7.



$$\frac{18 \cdot h}{2} = A$$

$$\frac{\ell \cdot s}{2} = \frac{1}{4}A = \frac{1}{4} \cdot \frac{18h}{2} = \frac{9h}{4} \Rightarrow \frac{h}{s} = \frac{4\ell}{2 \cdot 9} = \frac{2\ell}{9}$$

geometry $\frac{h}{s} = \frac{18}{\ell}$

so $\frac{h}{s} = \frac{18}{\ell} = \frac{2\ell}{9}$

$$\Rightarrow \frac{18 \cdot 9}{2} = \ell^2$$

$$9 = \ell^2$$

$$\underline{\ell = 9}$$

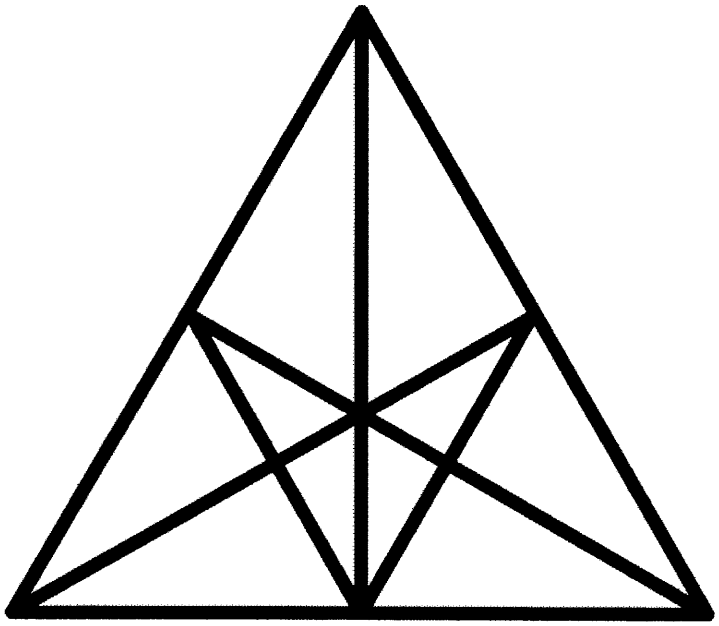
2012 PROBE Exam, Part II NAME: Key

1. One hour out of the station, the locomotive of a freight train develops trouble that slows its speed to 3/5 of its average speed up to the time of the failure. Continuing at this reduced speed, it reaches its destination two hours late. Had the trouble occurred 50 miles beyond, the delay would have been reduced by 40 minutes. What was the average speed of the train before the locomotive got into trouble?

2. Imagine the symbols below to sit on scales. Scale 1 and 2 are perfectly balanced.
   1. ♢ ♡ ↔ ♠ ♠ ♠ ♠
   2. ♠ ♠ ♢ ♢ ♡ ↔ ♡ ♡
   3. ♡ ♠ ♠ ↔ ?

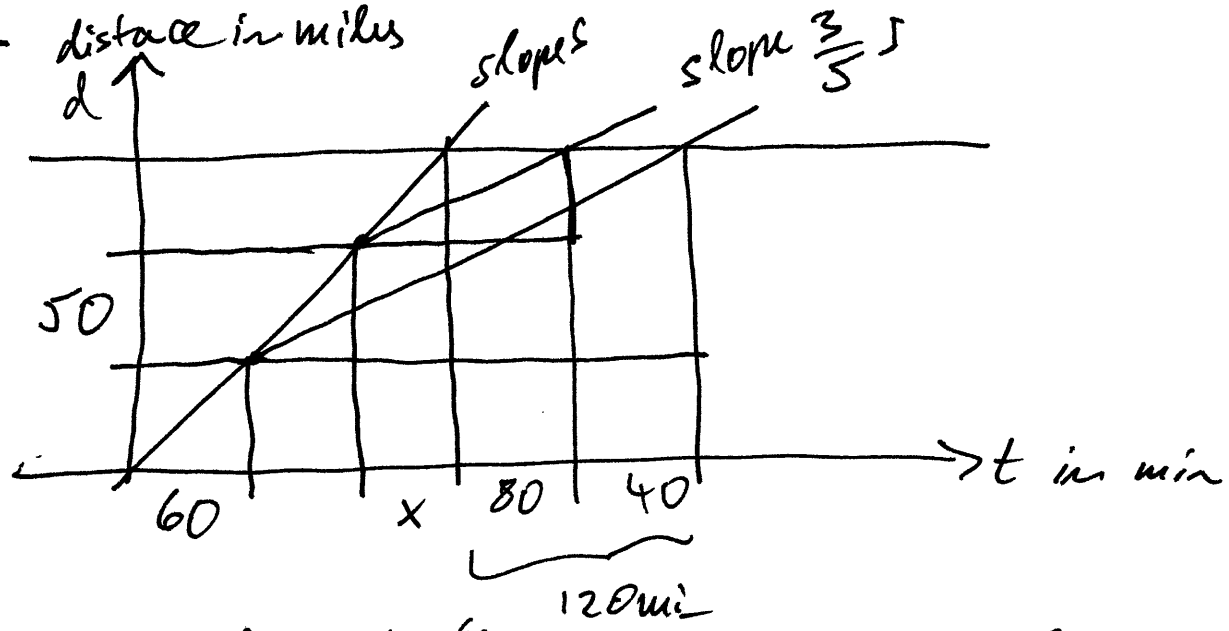
   How many ♢'s are needed to balance scale 3?

3. How many triangles?



Part II

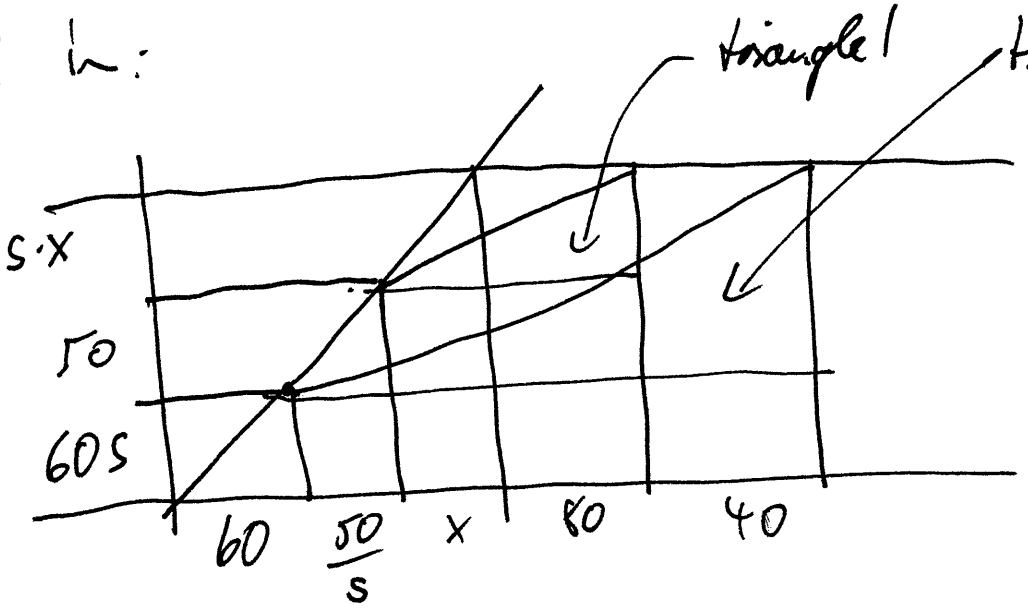
1.



s = speed originally, slow speed $= \frac{3}{5} \cdot s$

$d = s \cdot t$

fill in:



$\Delta 1$: $\frac{3}{5}s = \frac{sx}{80+x} \Rightarrow \frac{3}{5}(80+x) = x$

$\frac{240}{5} + \frac{3}{5}x = x$

$48 = \frac{2}{5}x$

$24 = \frac{1}{5}x$

$x = 120$

1 (cont)

triangle 2:

$$\frac{5 \cdot x + 50}{\frac{50}{s} + x + 120} = \frac{3}{5} s$$

$x = 120$

$$\frac{120s + 50}{\frac{50}{s} + 240} = \frac{3}{5} s$$

$$\frac{\left(120 + \frac{50}{s}\right) s}{240 + \frac{50}{s}} = \frac{3}{5} s$$

$$120 + \frac{50}{s} = \frac{3}{5}\left(240 + \frac{50}{s}\right) = 3\left(48 + \frac{10}{s}\right) = 144 + \frac{30}{s}$$

$$\frac{20}{s} = 24$$

$$s - \frac{20}{24} = \frac{10}{14} = \frac{5}{7} \ \frac{\text{miles}}{\text{minutes}}$$

$$= \frac{5}{7} \frac{\text{miles}}{\frac{1}{60}\text{ hr}} = \frac{60 \cdot 5}{7} \frac{\text{mi}}{\text{hr}} = \frac{300}{7} \frac{\text{mi}}{\text{hr}} \approx 42.8 \frac{\text{mi}}{\text{hr}}$$

Part II

2:

d = diamond
h = heart
s = spade

$1d + 1h = 4s$

$2d + 1h + 2s = 2h$

$\rightarrow h = 4s - d$

$1d + 1h - 4s = 0$

$+) \quad 2d - 1h + 2s = 0$

$3d \qquad - 2s = 0$

$3d = 2s$

$s = \frac{3}{2}d$

$1h + 2s = ?$

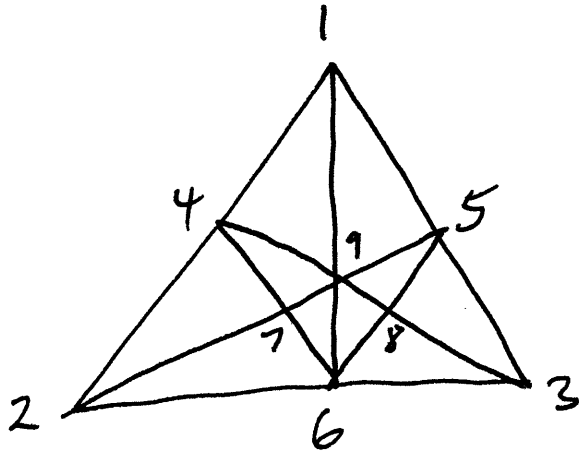
$= 4s - d + 2s$

$= 6s - d$

$= 6 \cdot \frac{3}{2}d - d$

$= 9d - d = \underline{8d}$

3:



proceed in lexicographic order:

123	234	346	468	567	679
125	235	356	469	569	689
126	239	358	479	589	
129	246	359			
134	247	368	(3)	(3)	(2)
136	249	369			
139	256	(6)			
146	267				
149	269				
156					
159	(9)				
(11)					

total:

11
9
6
3
3
2

34